Complex Dynamics and Chaos in Electronic Circuits

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DYNAMICAL SYSTEMS AND COMPLEXITY

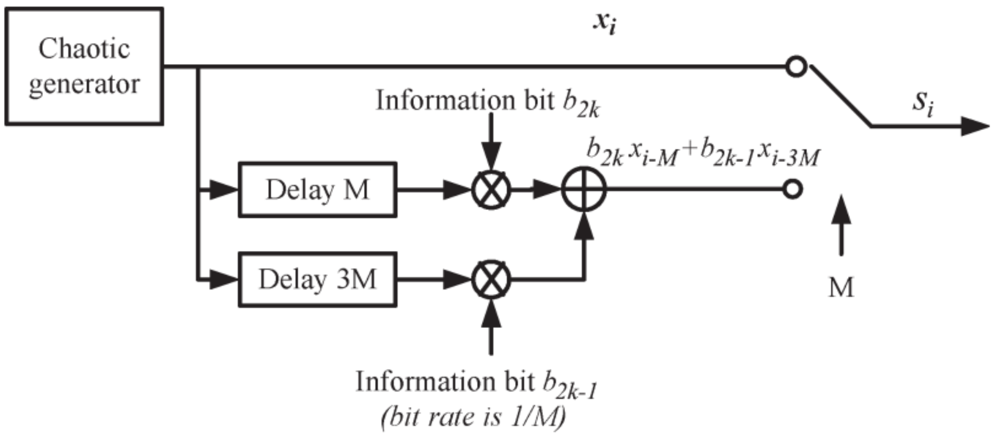
Chaos
In Electronic Circuits

Good or bad ?

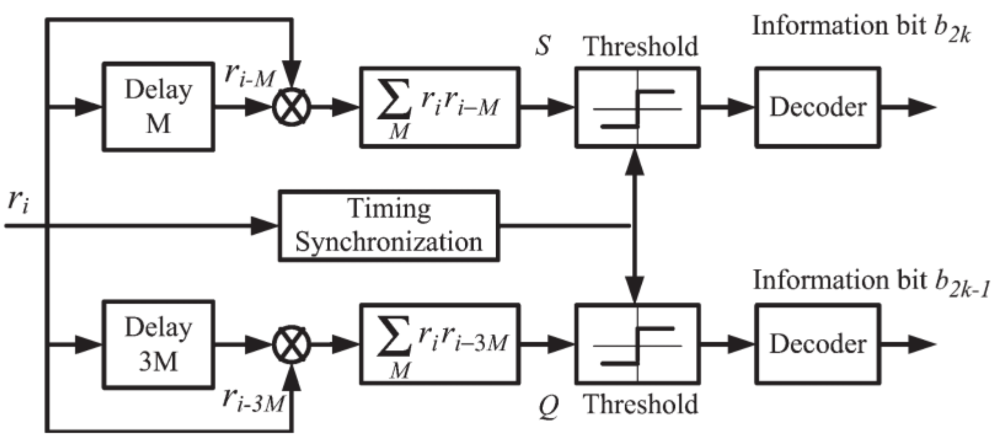
Lets examine a few cases …

Chaos Used in Communication Systems

Good



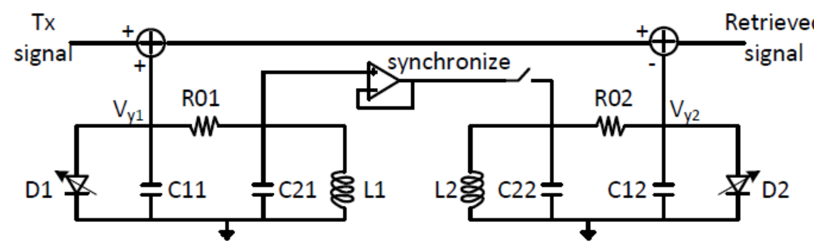
Modulator.



Receiver structure.

Chaotic Signal Used

- As a carrier
- For encryption / secure communication

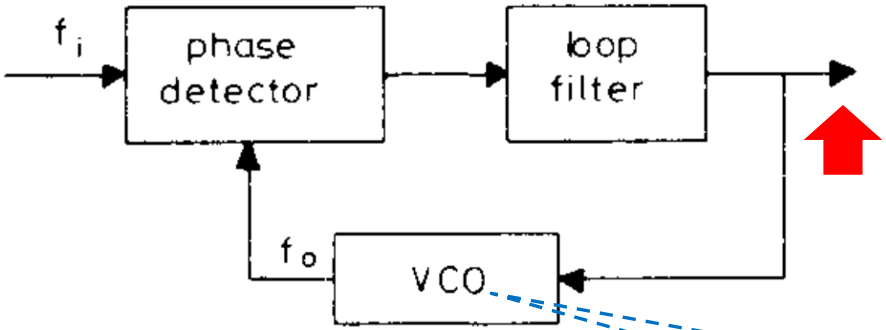


REF: Yang, Hua, and Guo-Ping Jiang. "High-efficiency differential-chaos-shift-keying scheme for chaos-based noncoherent communication." IEEE Transactions on Circuits and Systems II: Express Briefs 59.5 (2012): 312-316.

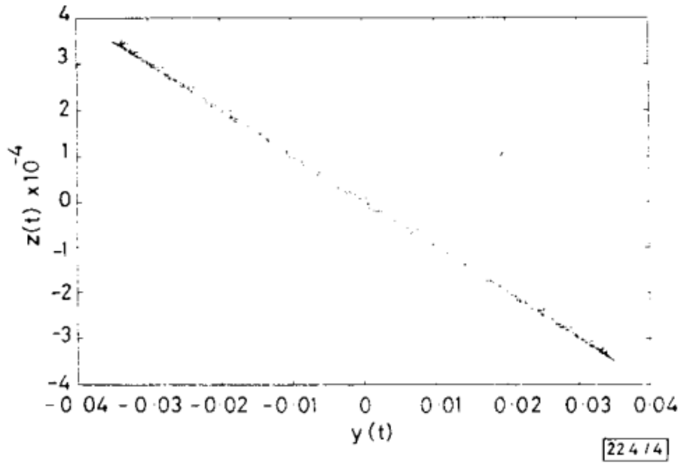
Chaos Appearing In
Phase-Locked Loops (PLL)

Bad

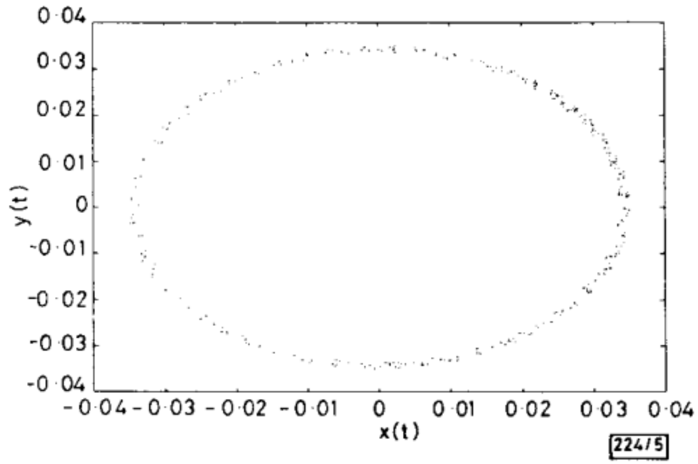
IN: $\sin(2\pi f_i t)$ OUT: $\sin(2\pi K f_i t)$



(**V**oltage **C**ontrolled **O**scillator)



Projection onto (y, z) plane



Projection onto (x, y) plane

If Chaotic behavior appears

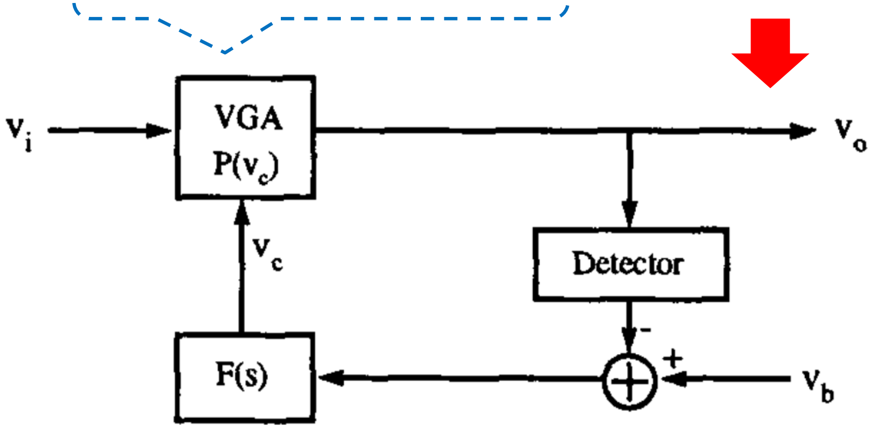
- It contaminates the output signal with "noise"
- It is probably catastrophic for the operation of the system the PLL is part of

REF: Chou, J. H., Y. H. Chu, and S. Chang. "Chaos in phase-locked loops." *Electronics letters* 27.9 (1991): 750-751.

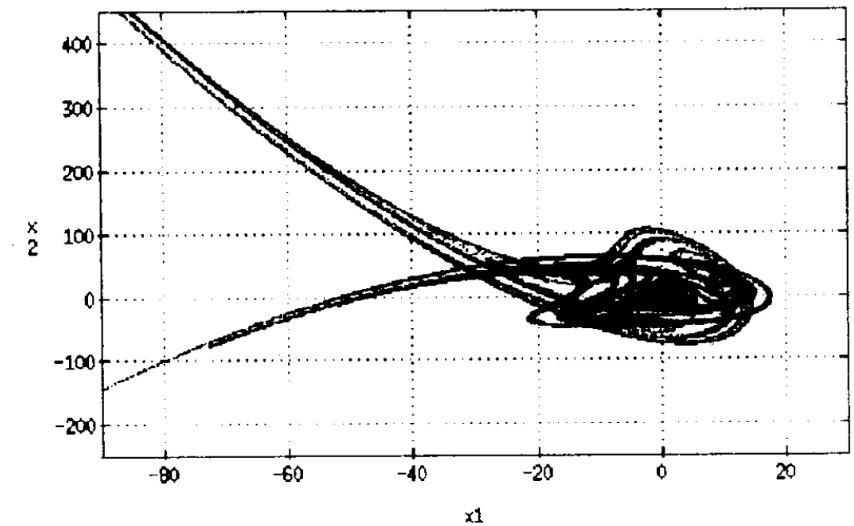
Chaos Appearing In
Automatic Gain Control (AGC) Loops

Bad

(**V**ariable **G**ain **A**mplifier)



Block diagram of an automatic gain control loop.



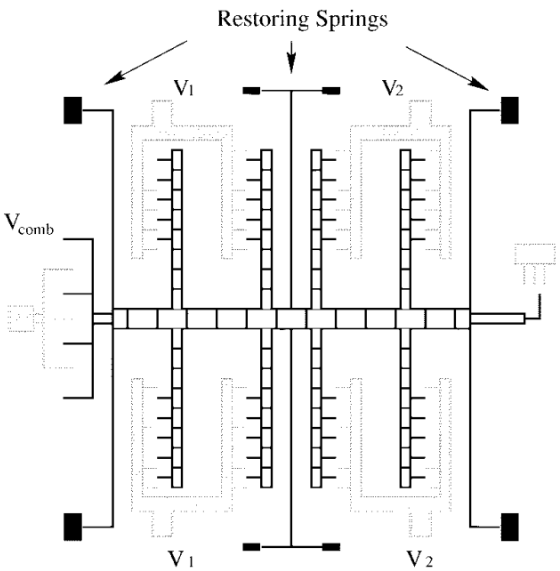
If Chaotic behavior appears

- It contaminates the output signal with multiplicative “noise”
- It may be catastrophic for the operation of the system the VGA is part of

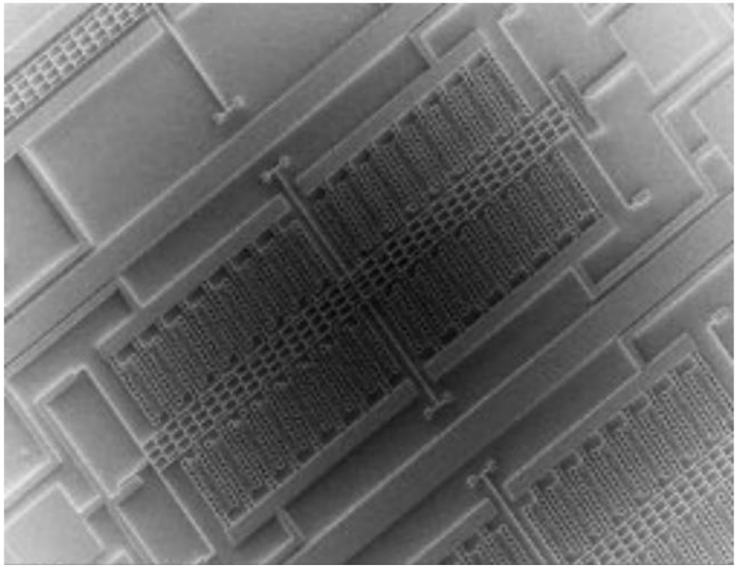
<u>REF</u>: Chang, F-J., S-H. Twu, and S. H. Y. A. N. G. Chang. "Global bifurcation and chaos from automatic gain control loops." *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications* 40.6 (1993): 403-412.

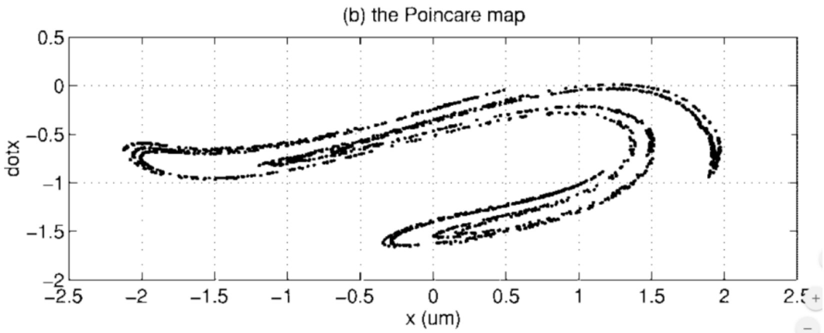
Chaos Appearing In Micro-Electro-Mechanical (MEM) Devices

Good or Bad



The schematic diagram of the parallel-reduction tunable oscillator.



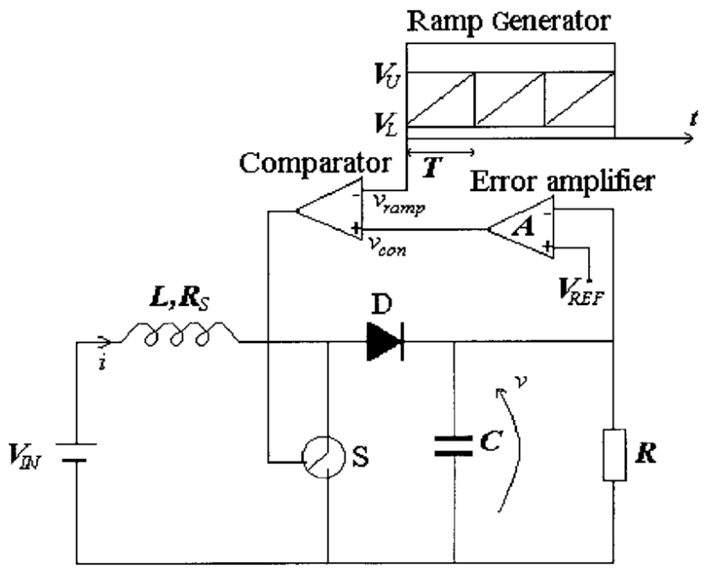


Chaotic behavior may be deliberate or accidental

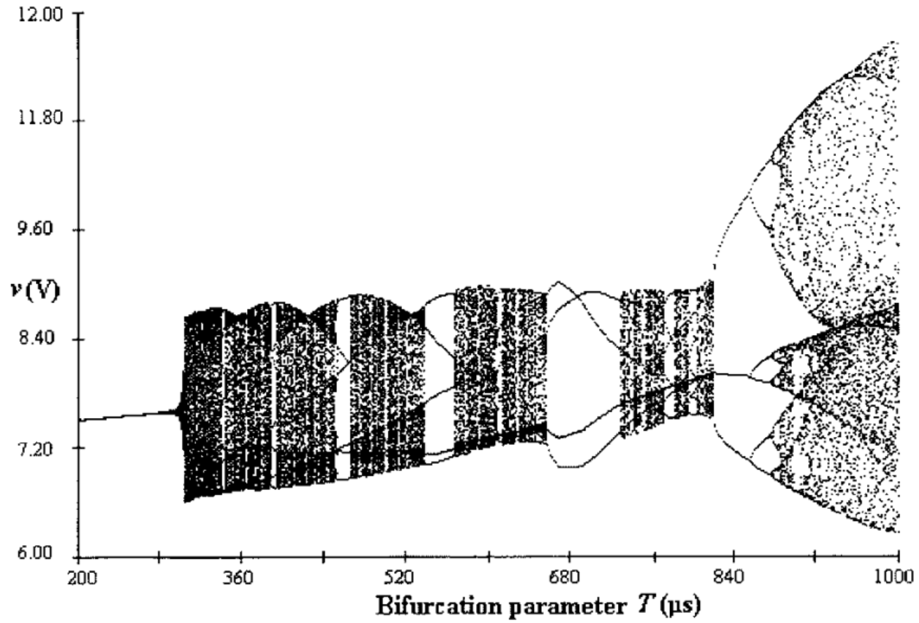
- bad / Good depends on Application
- In the REF a Chaotic generator is presented

REF: Wang, Yongmei Cindy, et al. "Chaos in MEMS, parameter estimation and its potential application." IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications 45.10 (1998): 1013-1020.

Chaos Appearing In
DC to DC Converters



Block diagram of the PWM controlled boost converter.



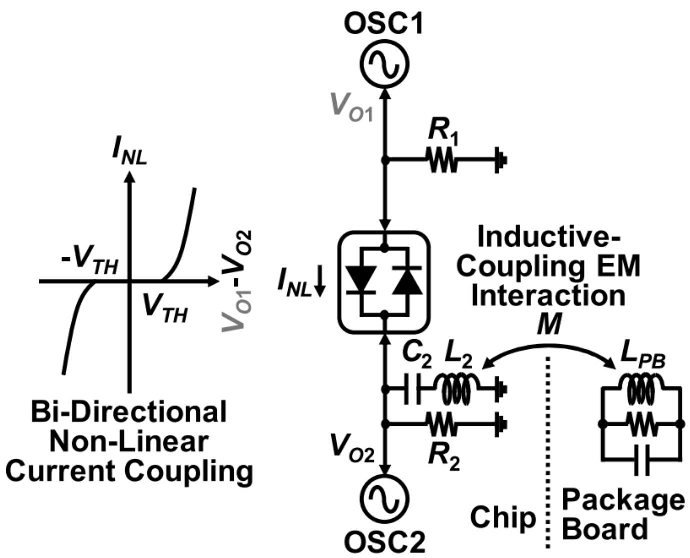
If Chaotic behavior appears (with the exception of deliberate chaotic behavior for spectrum spreading)

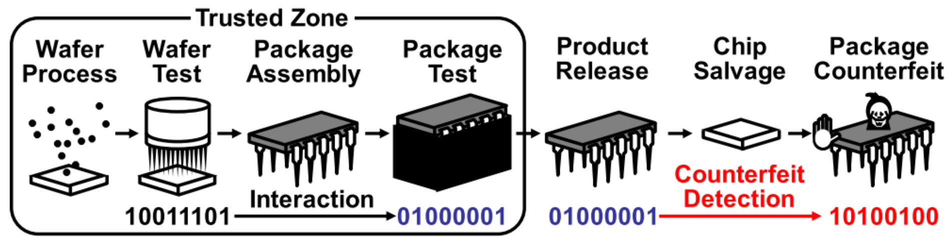
- Will contaminate supply voltage with "noise"
- If oscillation too large it may damage the converter / the circuits it powers / the power source

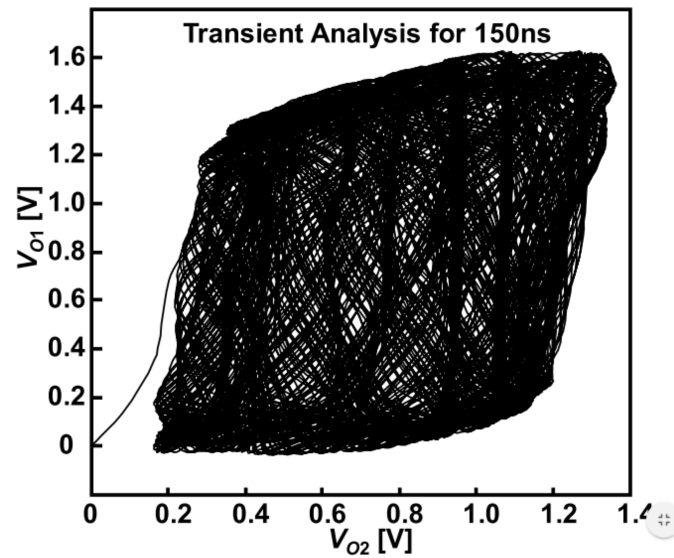
REF: El Aroudi, Abdelali, et al. "Hopf bifurcation and chaos from torus breakdown in a PWM voltage-controlled DC-DC boost converter." IEEE Trans. on Cir. and Syst. I: Fundamental Theory and Applications 46.11 (1999): 1374-1382.

Chaos Used In
Counterfeiting Counter-measures

Good







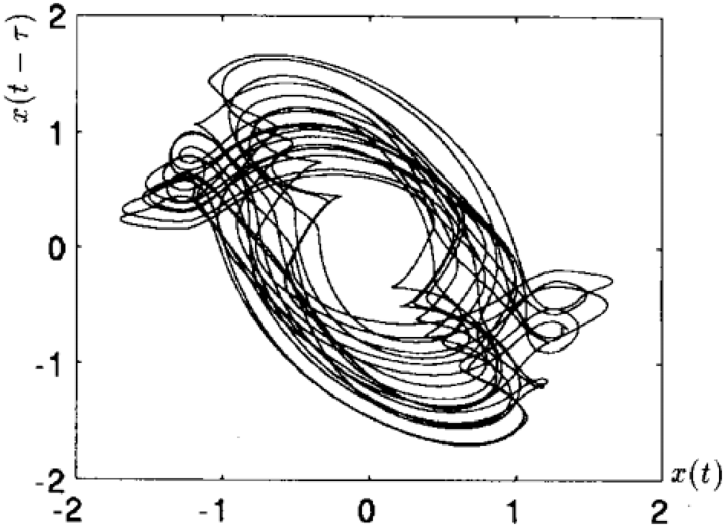
Use Chaotic behavior to

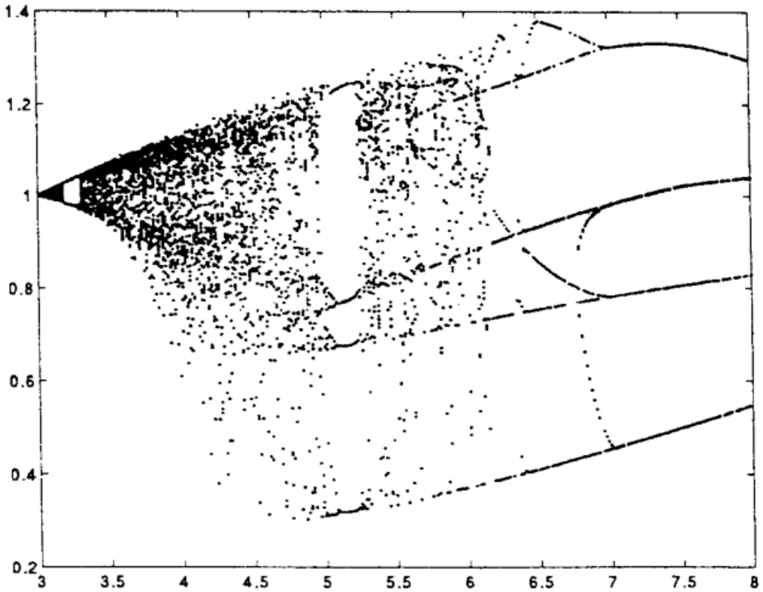
- Create a specific ID for each Chip + Package + PCB
- Use ID against malicious counterfeiting

REF: Miura, Noriyuki, et al. "Chip-Package-Board Interactive PUF Utilizing Coupled Chaos Oscillators With Inductor." IEEE Journal of Solid-State Circuits 53.10 (2018): 2889-2897.

Chaos Appearing In
Certain Classed of Neural Networks

Good or Bad





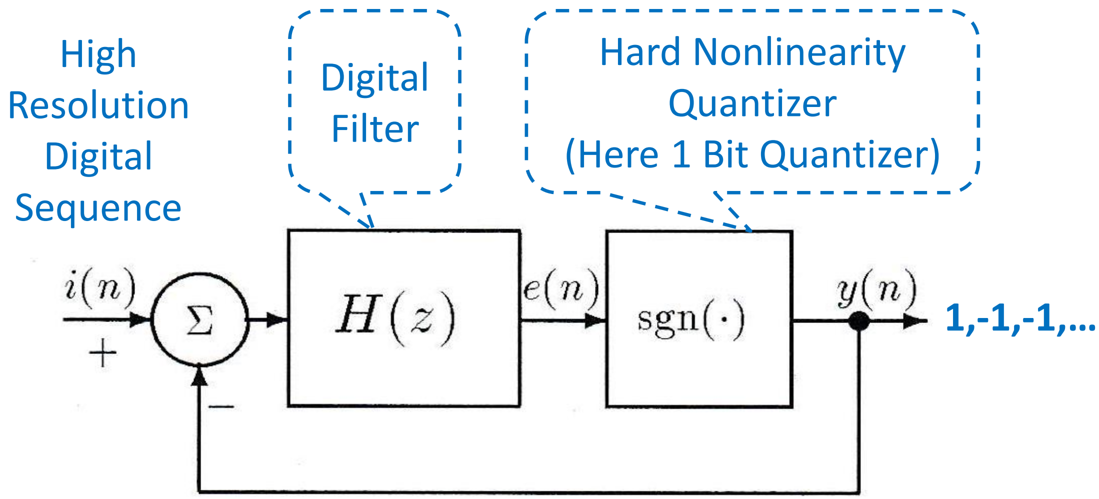
Chaotic behavior can

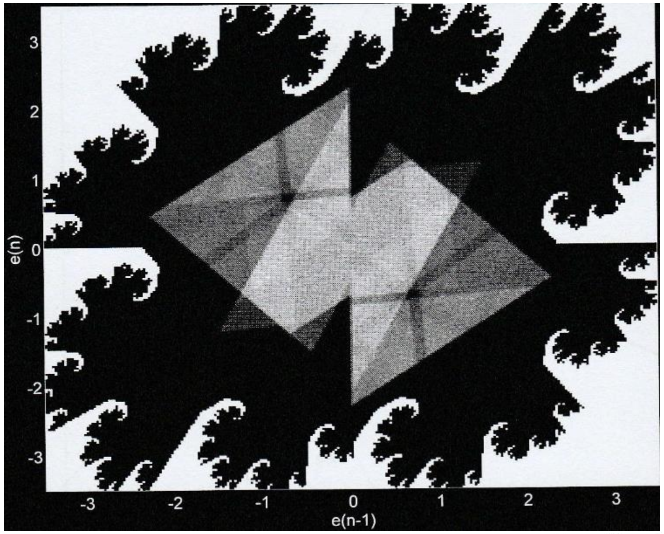
- Work as perturbation leading to a better local optimum
- Or, can lead to instability

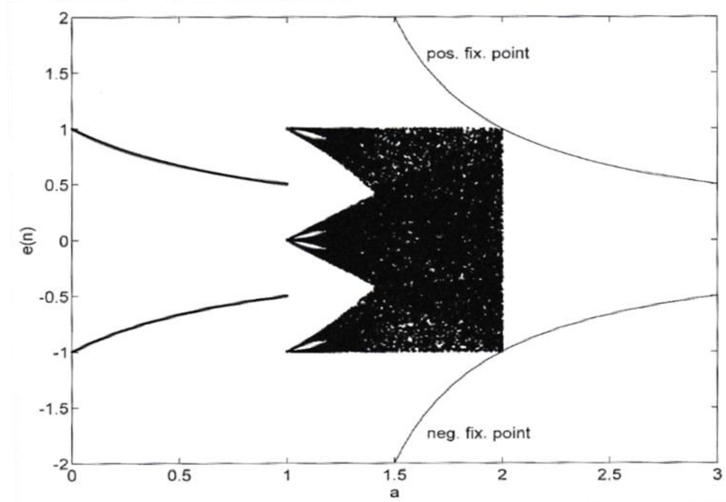
REF: Lu, Hongtao, Yongbao He, and Zhenya He. "A chaos-generator: Analyses of complex dynamics of a cell equation in delayed cellular neural networks." IEEE Transcactions. on Circuits and Systems I: 45.2 (1998): 178-181.

Chaos Appearing In
Sigma-Delta Modulators

Good







Chaotic behavior is "desirable" as part of the goal to

- To "break" periodic patterns (work as random dithering)
- "Spread" the PSD of the quantization noise

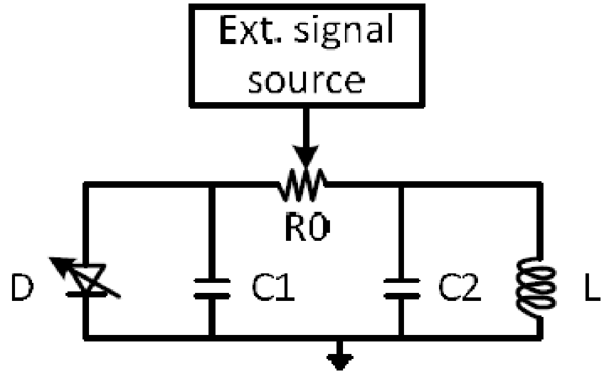
  (outside of the signal's Bandwidth)

<u>REF</u>: Risbo, Lars, and John Aasted Sørensen. "Sigma-delta modulators-stability analysis and optimization." (1995).

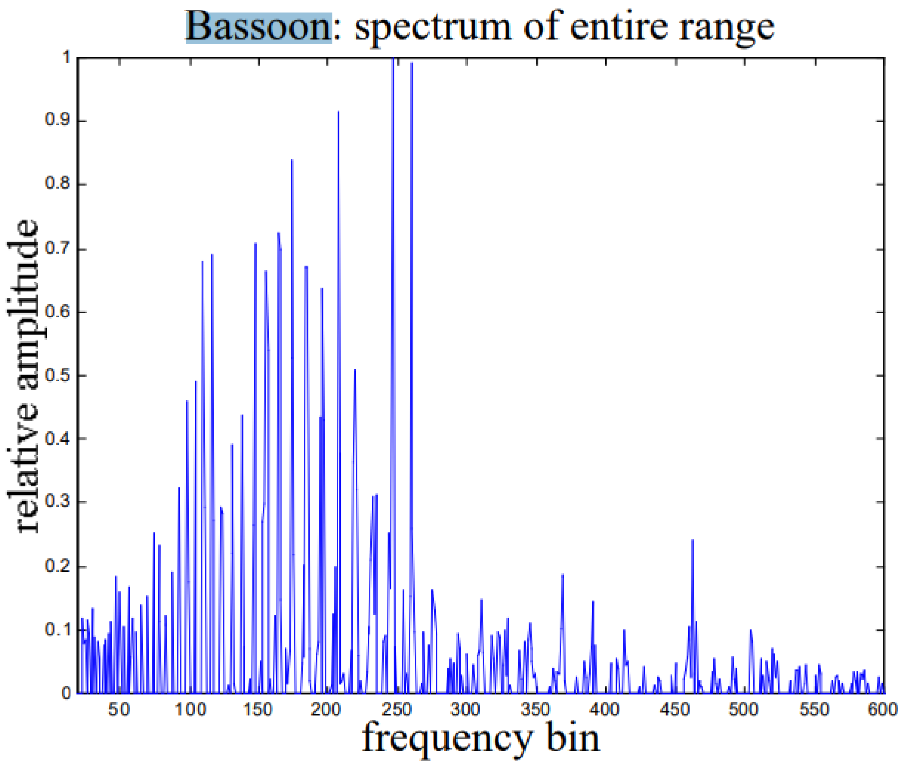
Chaos Used In
Sound Synthesis

Good

Use of Chua's circuit



(Φαγκότο)



Use Chaotic behavior to

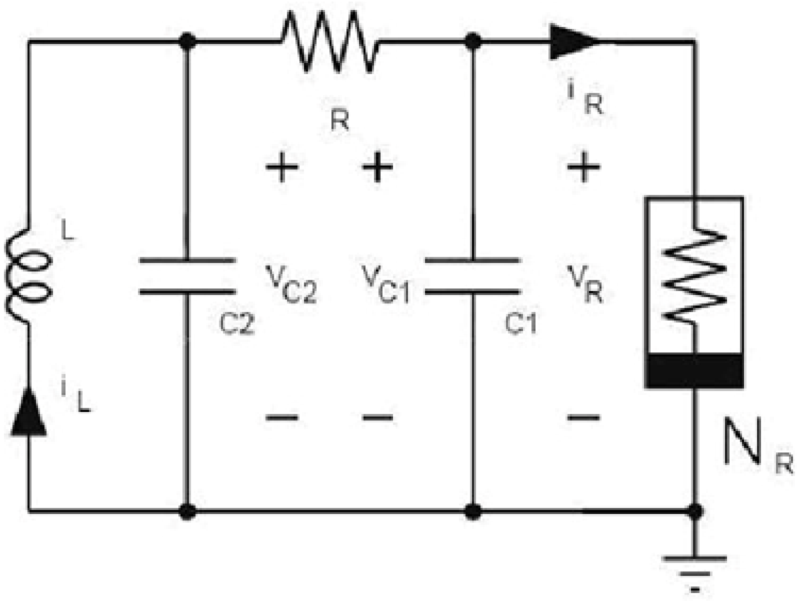
- Synthesize instrument sounds
- Synthesize nature's sounds

REF: Johnson, Kimo, and NEW HAMPSHIRE DURHAM. "Controlled chaos and other sound synthesis techniques", BS degree Thesis(2000).

Chua's CHAOTIC Circuit

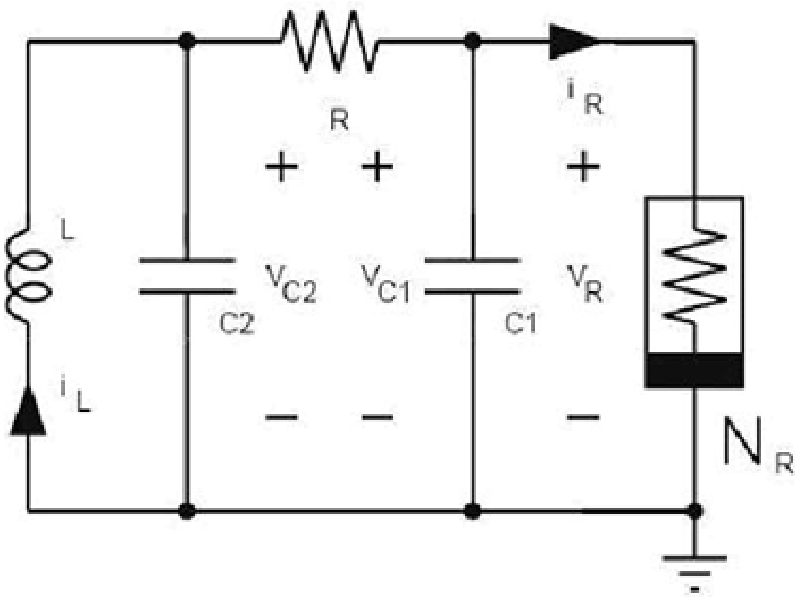
A Practical View

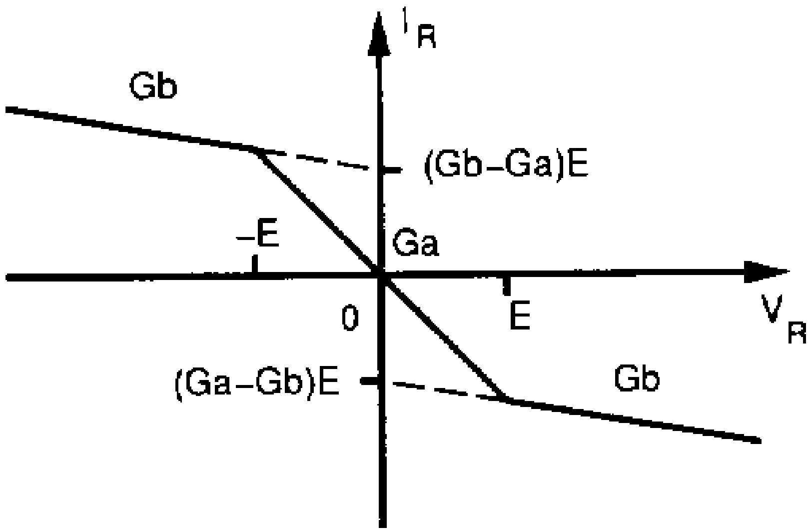
Chua's Chaotic Circuit



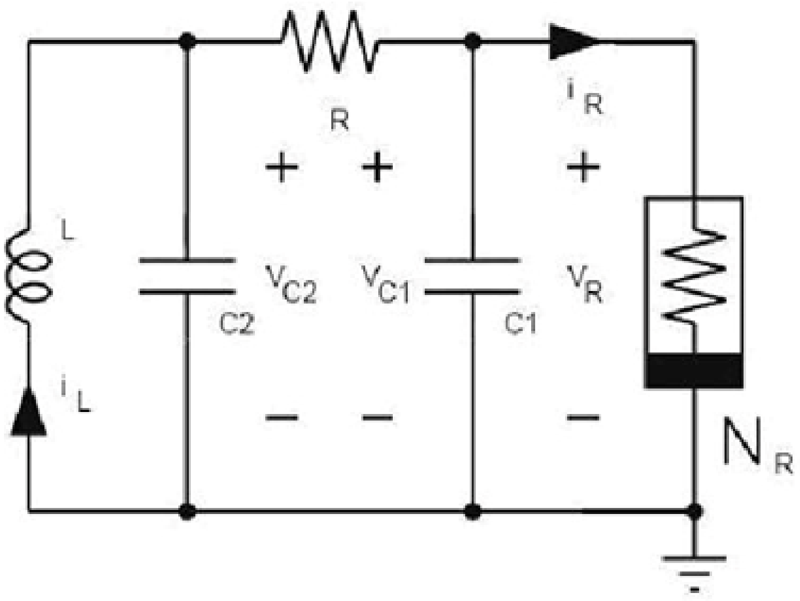
1. The basic circuit consists of :
   - two capacitors C1, C2
   - one inductor L
   - one resistor R
   - one nonlinear element – resistor NR.
2. The parallel combination of C2 and L constitutes a lossless resonant circuit. The resistance R provides the coupling between this, the active nonlinear resistor NR, and C1.
3. Resistance R is a trimming parameter of the circuit

Chua's Chaotic Circuit
Nonlinear Resistance





State Differential Equations

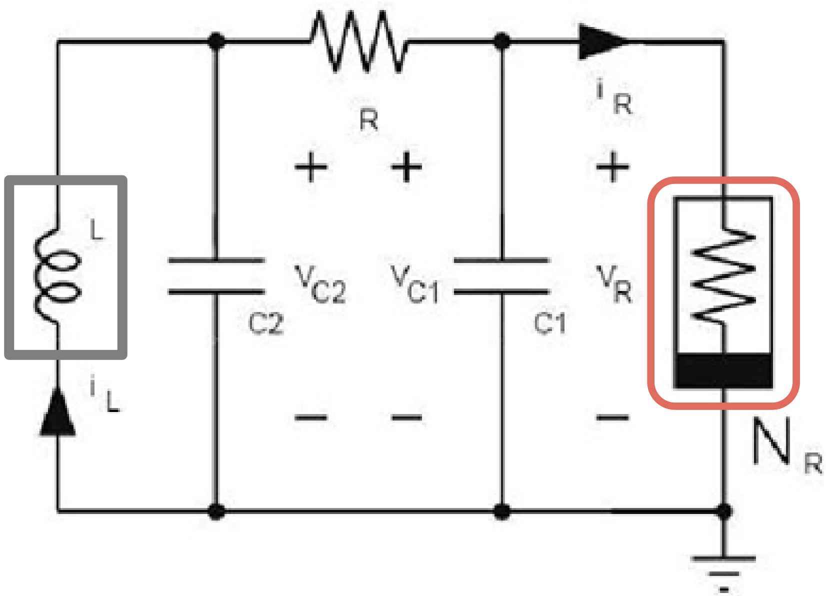


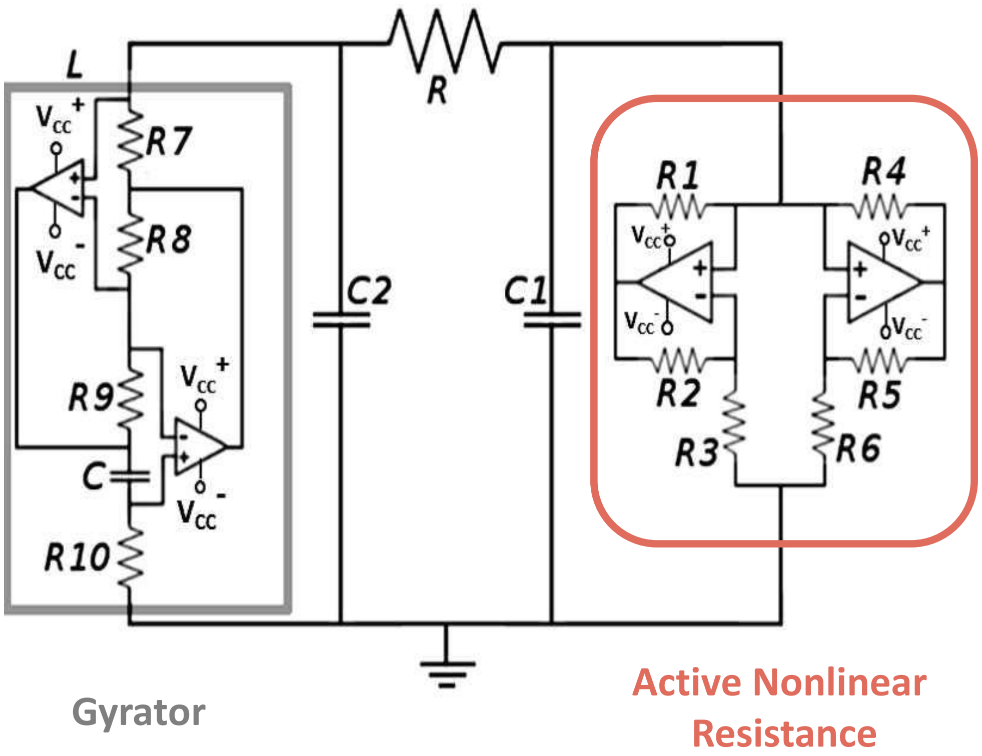
$$C_1 \frac{dV_{C1}}{dt} = \frac{V_{C2} - V_{C1}}{R} - f(V_{C1})$$

$$C_2 \frac{dV_{C2}}{dt} = \frac{V_{C1} - V_{C2}}{R} + i_L$$

$$L \frac{di_L}{dt} = -V_{C2}$$

Practical Implementation

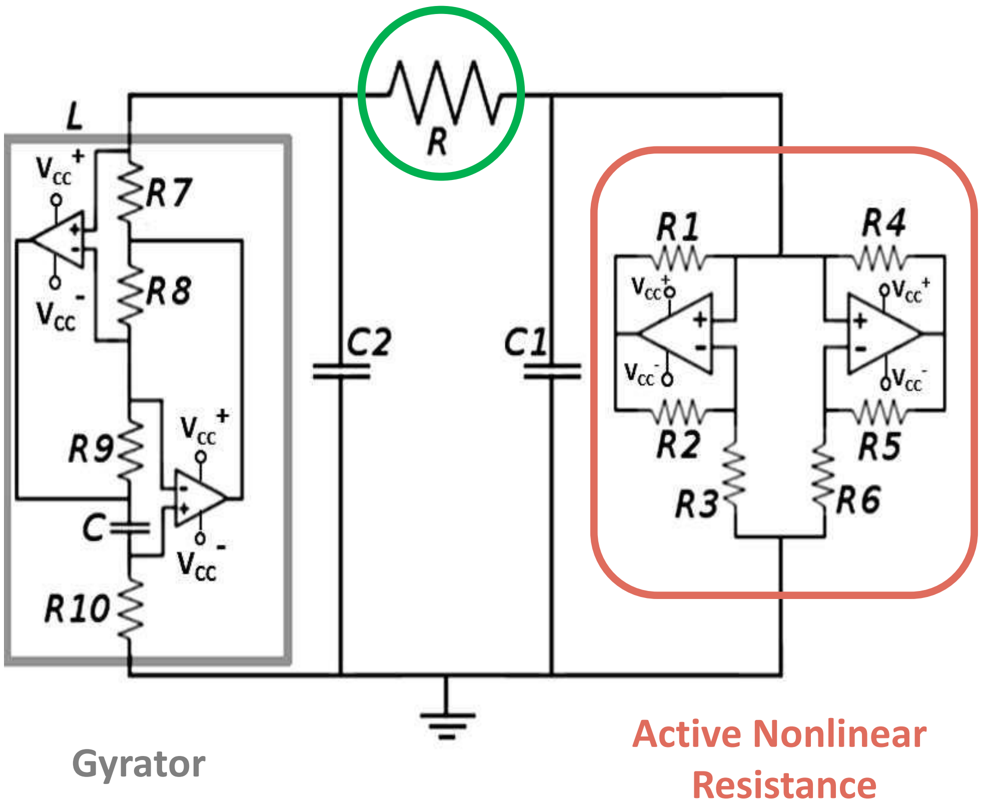




$$L = \frac{R_7 R_9 R_{10} C}{R_8}$$

Practical Implementation
Typical Component Sizes

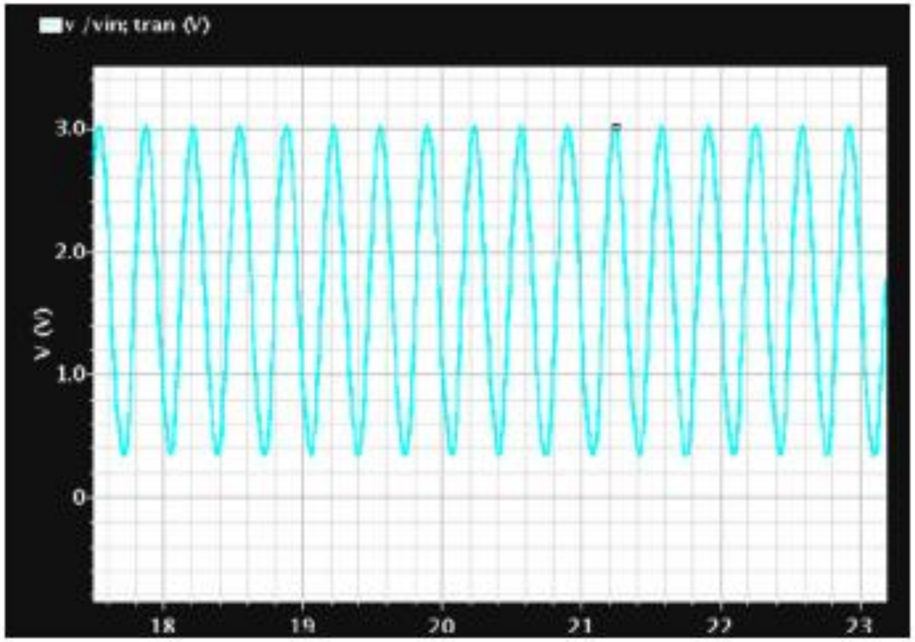
R=2.2 kΩ (pot.)	C=100 nF
R_1=220 Ω	C_1=10 nF
R_2=220 Ω	C_2=100 nF
R_3=2.2 kΩ	
R_4=22.0 kΩ	L=15 mH
R_5=22.0 kΩ	
R_6=3.3 kΩ	
R_7=100 Ω	
R_8=1.0 kΩ	
R_9=1.0 kΩ	
R_{10}=10 kΩ (pot.)	

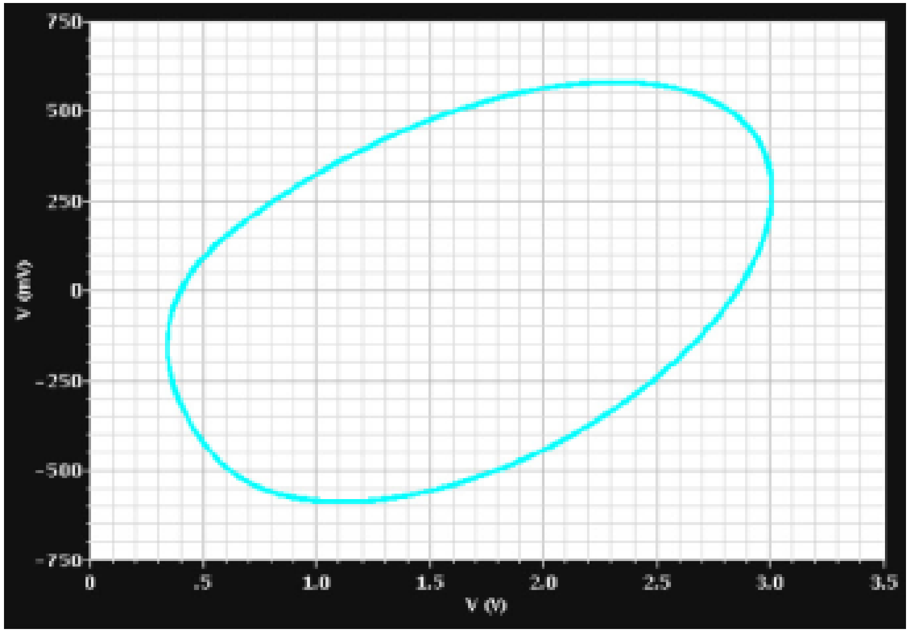


Gyrator: Passive, Linear, Lossless, 2-Port Network
Can emulate inductive behavior using only capacitors

$$L = \frac{R_7 R_9 R_{10} C}{R_8}$$

Transient and Phase Space
R = 1.90 kΩ

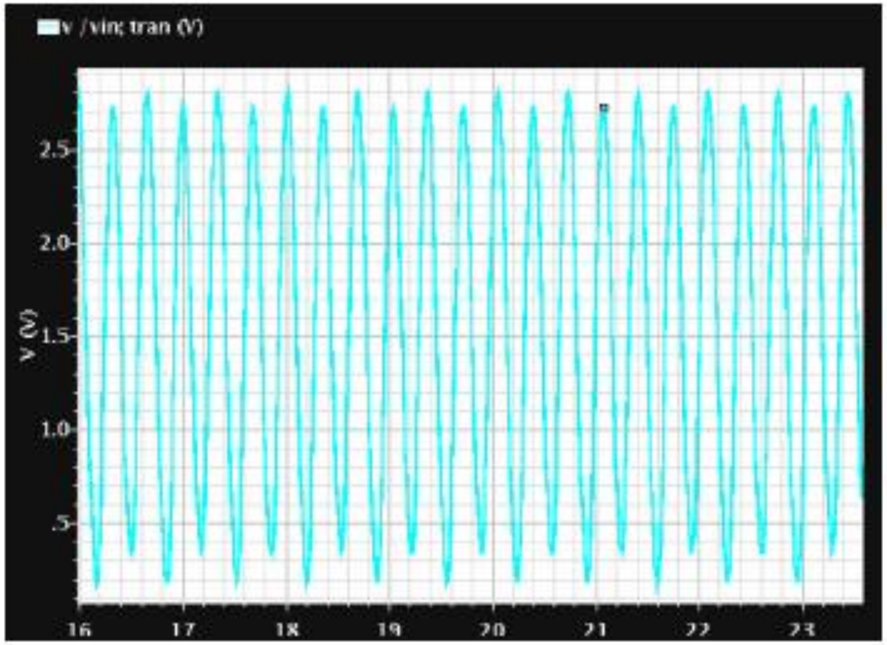


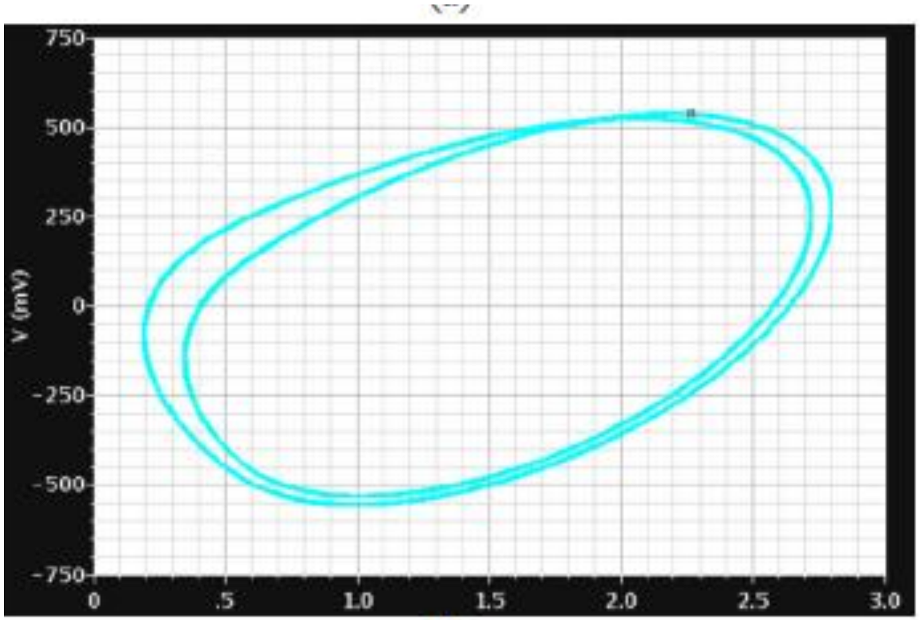


- The transient waveform has **one** oscillation cycle
- The trajectory in phase space encircles the attractor **once**.

Transient and Phase Space
R = 1.87 kΩ

[Circuit]
[Simulation]

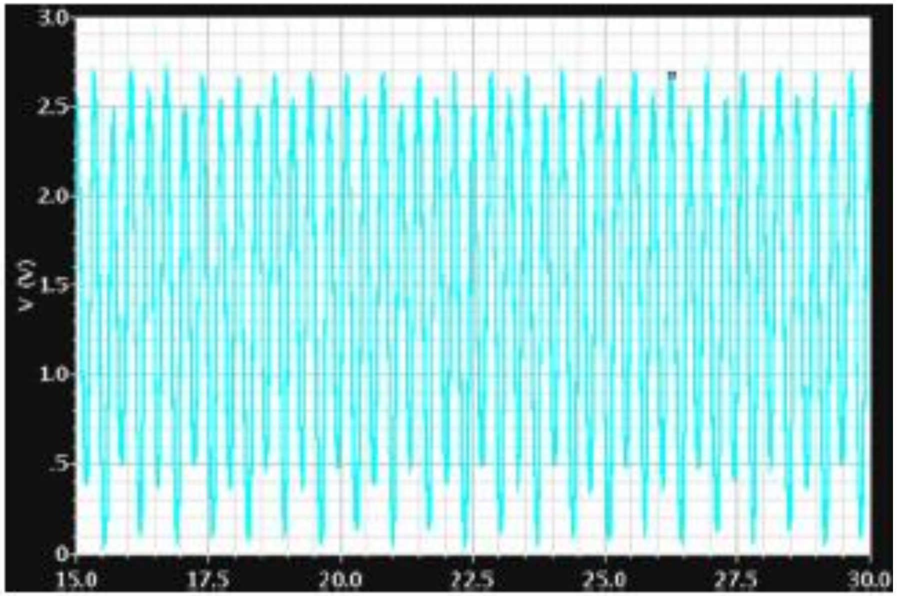




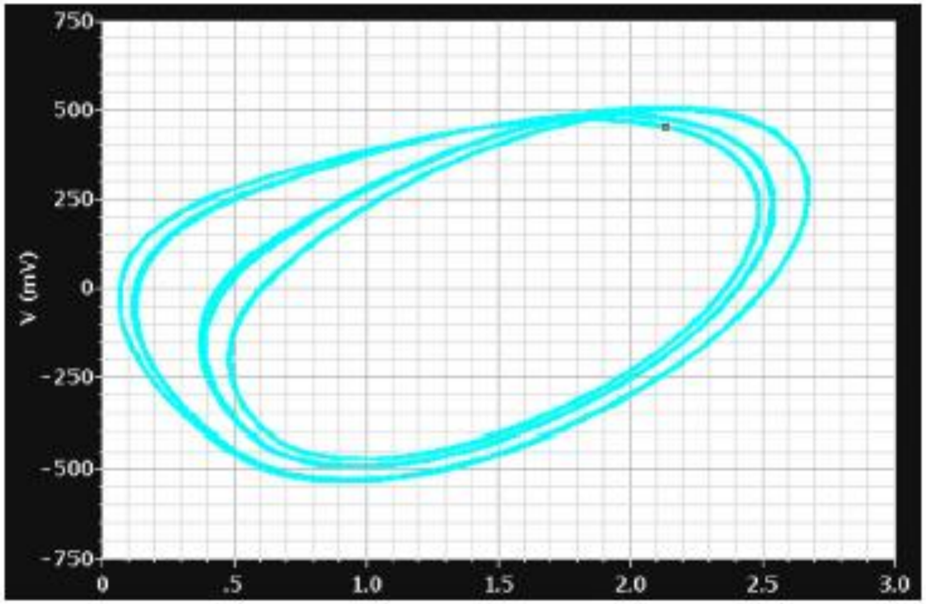
- The transient waveform has **two** oscillation cycles
- The trajectory in phase space encircles the attractor **twice**

Transient and Phase Space
R = 1.85 kΩ

[Circuit]
[Simulation]



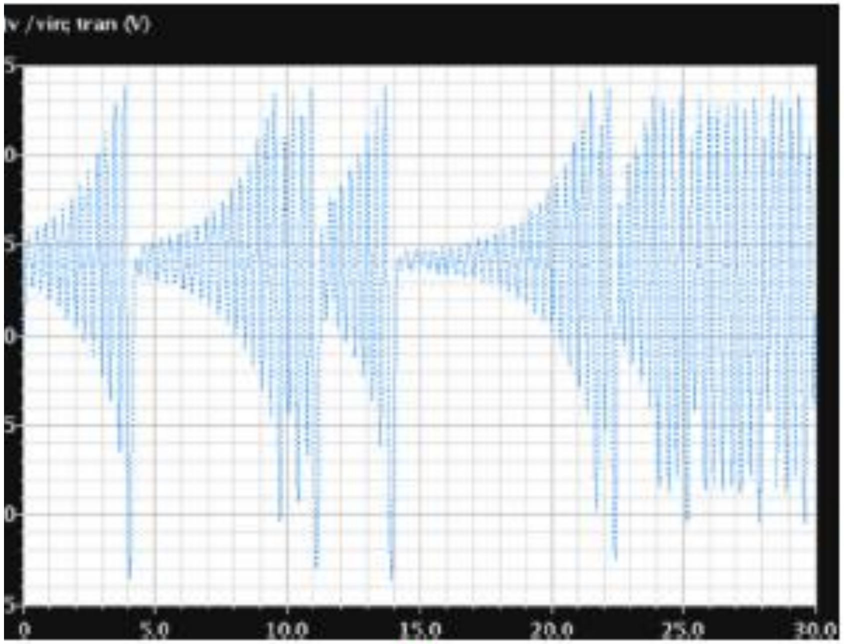
(a)

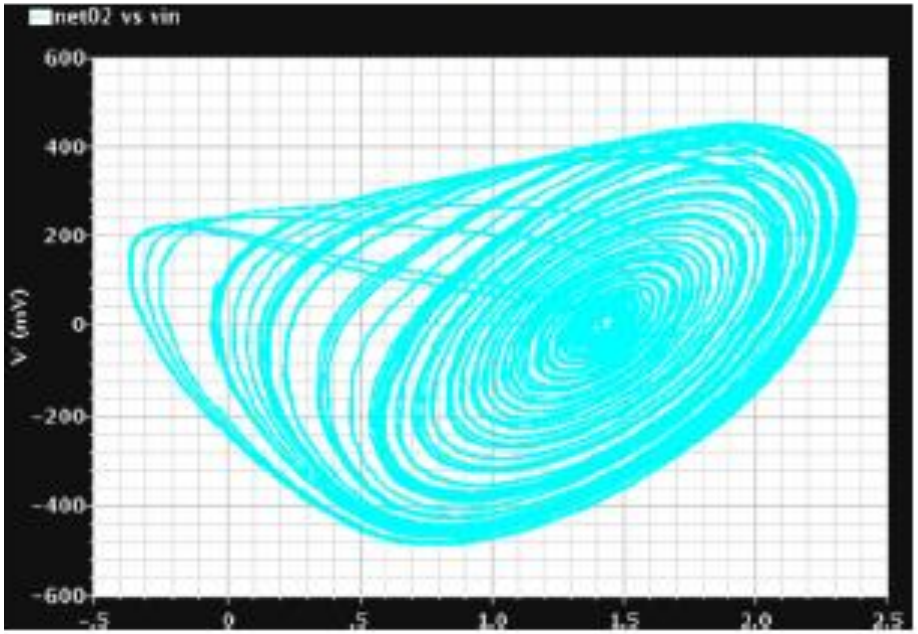


(b)

- The transient waveform has **four** oscillation cycles
- The trajectory in phase space encircles the attractor **four times**

Transient and Phase Space
R = 1.80 kΩ

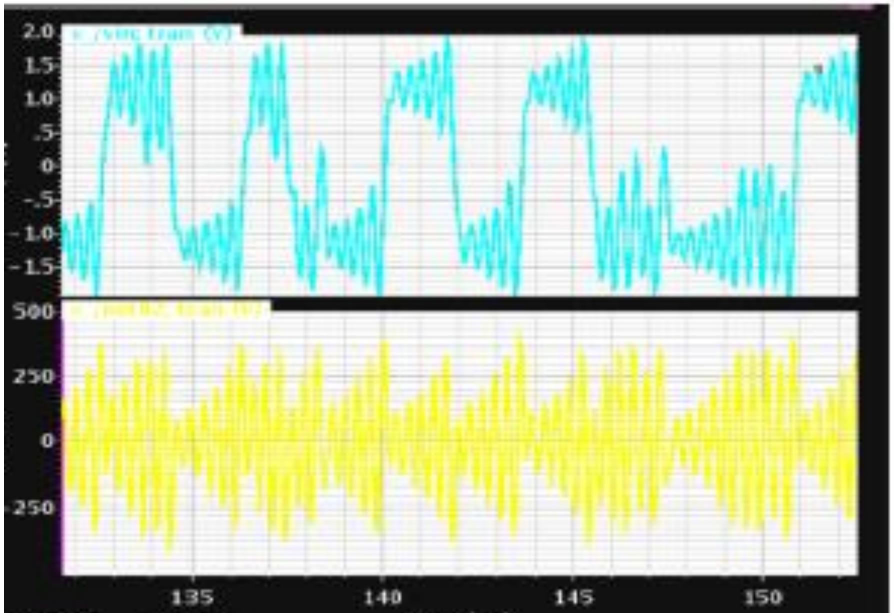


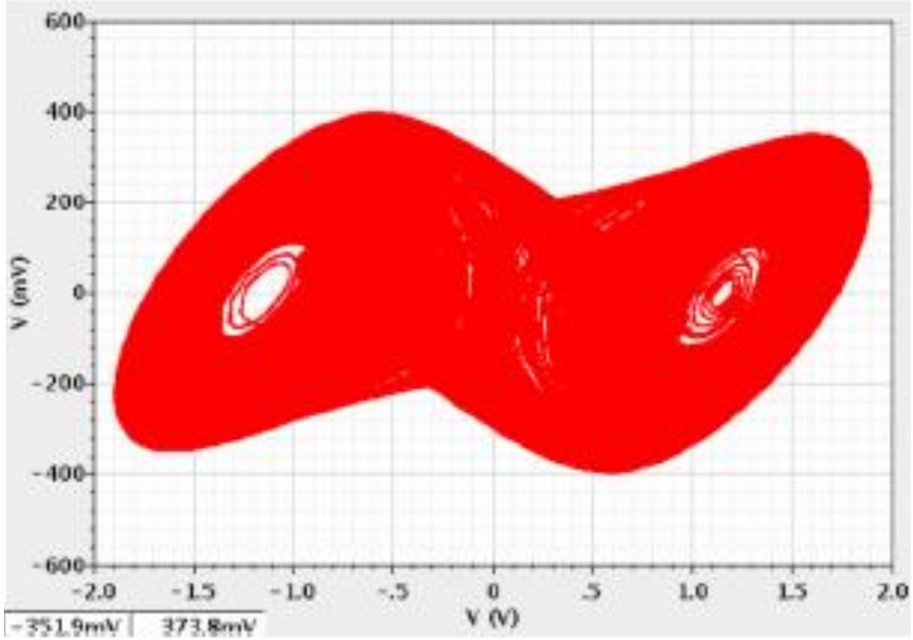


- The transient waveform is **not** periodic
- The trajectory in phase space encircles the attractor **infinite times**

Transient and Phase Space
R = 1.70 kΩ

[Circuit]
[Simulation]

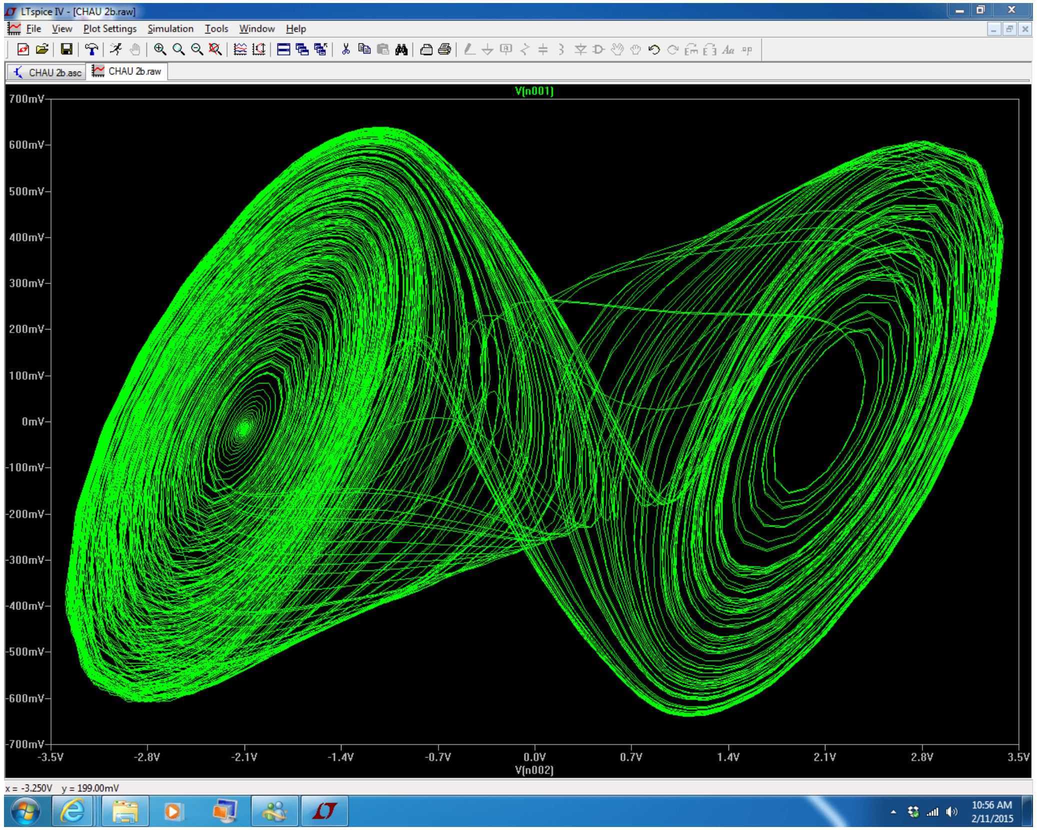




- The attractor becomes double-scroll
- The transient waveforms of Vc2(upper part) and Vc1(lower part) are shown

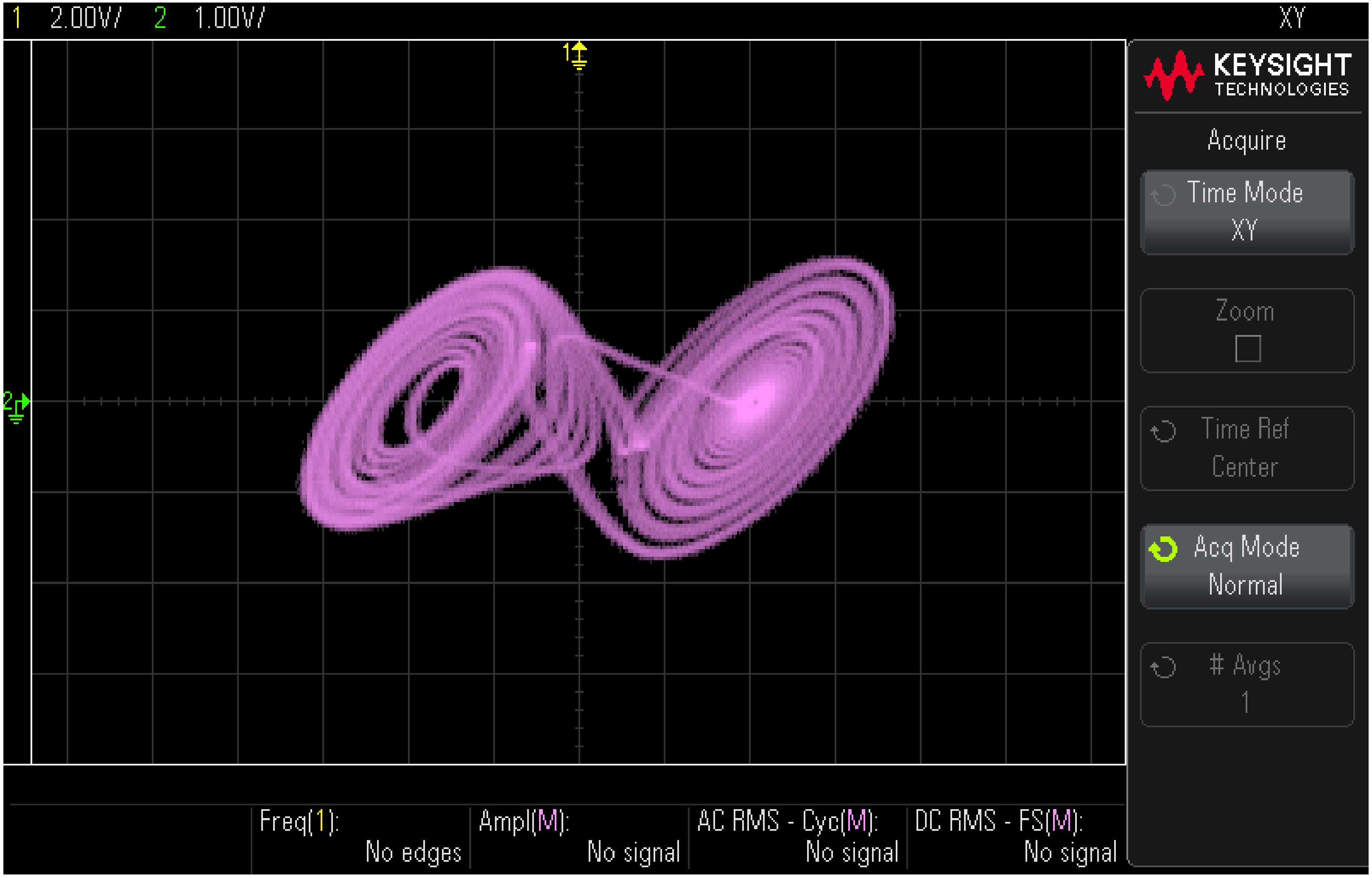
Phase Space
Chaos : Simulation

[Circuit]
[Simulation]

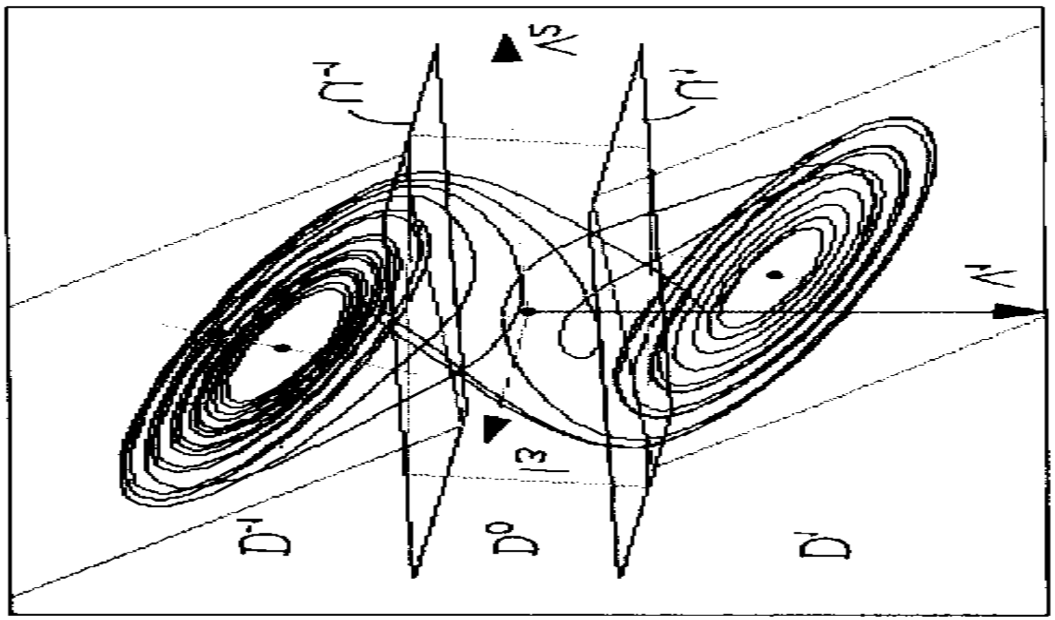


Phase Space
Chaos : Measurement

[Circuit]
[Measurements]

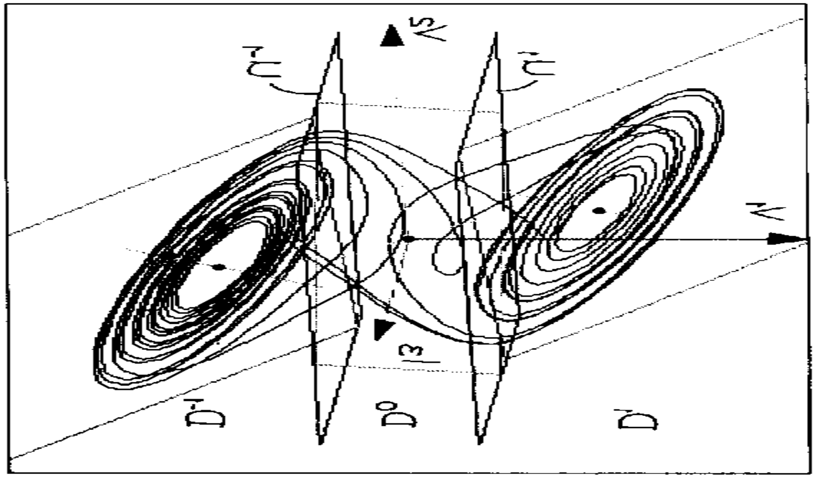


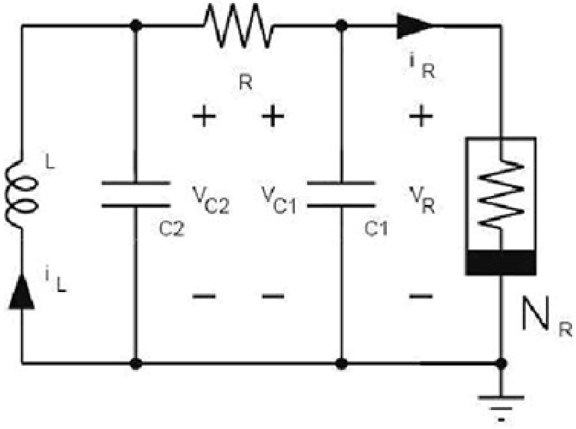
Phase Space
Chaos : Theory



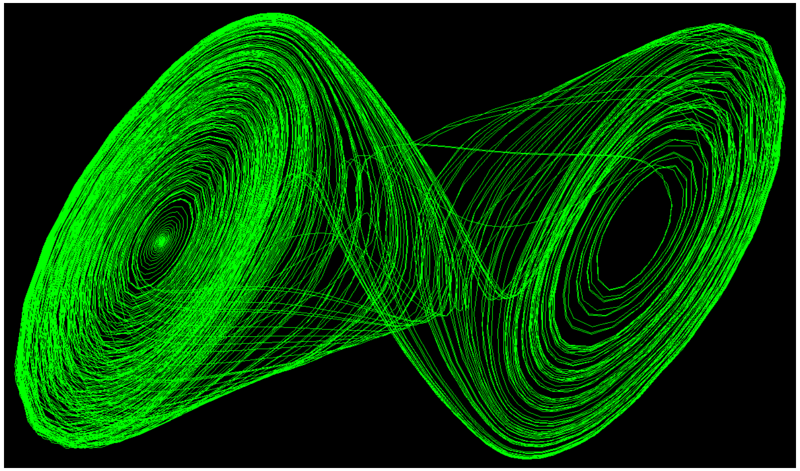
Phase Space
Chua's Chaos: Theory, Simulation, Measurements

Theory

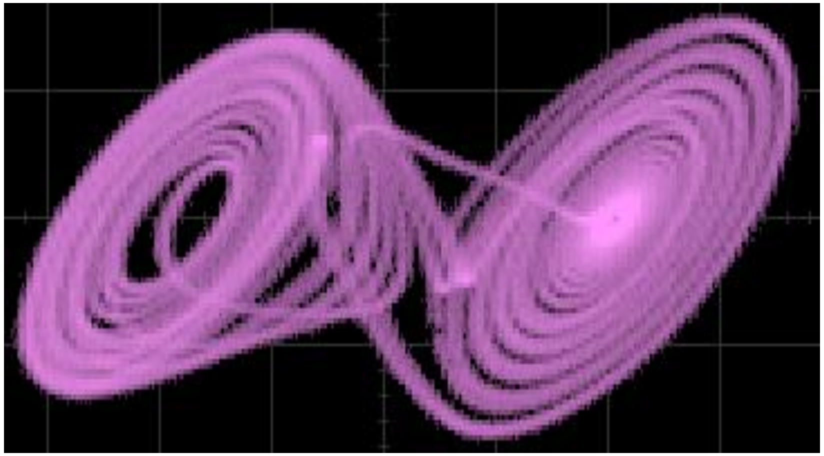




Simulation



Measurements



Demonstration

Chua’s CHAOTIC Circuit

LIVE!